9. Magnetic Field II: sources

Tipler eta Mosca chapters: 26 & 27

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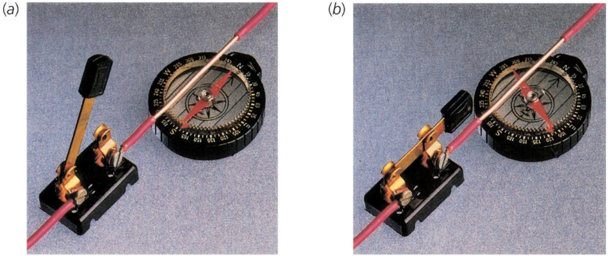
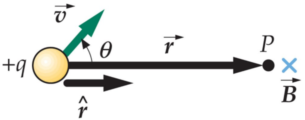


Sources of the Magnetic Field: Biot-Savart Law

When a point charge q moves with velocity $\vec{v}$, the moving point charge produces a magnetic field $\vec{B}$ in space, given by:

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}}$$

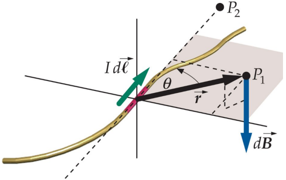
$$\mu_0 = 4\pi \times 10^{-7}\mathrm{N/A}^2$$



In the previous chapter we extended our discussion of forces on point charges to forces on current elements by replacing $q\vec{v}$ with the current element $Id\vec{l}$. We do the same for the magnetic field produced by a current element:

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}}$$

Biot-Savart law

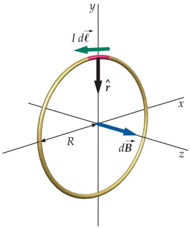


Biot-Savart application: $\vec{B}$ Due to a Current Loop

a) At the center of the loop:

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin(90)}{R^2} \rightarrow B = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int dl = \frac{\mu_0}{4\pi}\frac{I}{R^2}2\pi R$$

$$B = \frac{\mu_0 I}{2R}$$



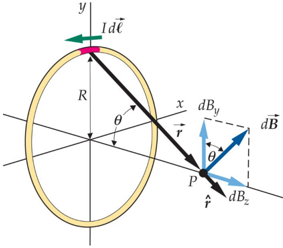
b) At P on the axe of the loop:

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{I|d\vec{l}\times\hat{r}|}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl}{(z^2+R^2)}$$

$$dB_z = dB\sin\theta = \left(\frac{\mu_0}{4\pi}\frac{Idl}{(z^2+R^2)}\right)\left(\frac{R}{\sqrt{z^2+R^2}}\right)$$

$$B_z = \int dB_z = \int \frac{\mu_0}{4\pi}\frac{IRdl}{(z^2+R^2)^{3/2}} = \frac{\mu_0 IR}{4\pi(z^2+R^2)^{3/2}}\int dl$$

$$\boxed{B_z = \frac{\mu_0}{2}\frac{R^2 I}{(z^2+R^2)^{3/2}}}$$



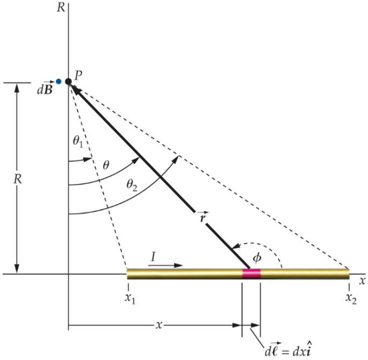
Biot-Savart application: $\vec{B}$ Due to a Current in a Straight Wire

$$dB = \frac{\mu_0}{4\pi}\frac{Idx}{r^2}\sin\phi \quad \text{or} \quad dB = \frac{\mu_0}{4\pi}\frac{Idx}{r^2}\cos\theta$$

$$x = R\tan\theta \rightarrow dx = R\frac{d\theta}{\cos^2\theta} \quad \text{and} \quad r = \frac{R}{\cos\theta}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\cancel{R}d\theta}{\cancel{\cos^2\theta}}\frac{\cancel{\cos^2\theta}}{R^{\cancel{2}}}\cos\theta$$

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}\int_{\theta_1}^{\theta_2}\cos\theta d\theta = \boxed{\frac{\mu_0}{4\pi}\frac{I}{R}(\sin\theta_2 - \sin\theta_1)}$$

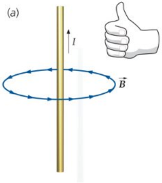


Particular cases:

a) Infinitely long:

$\theta_1 = -90^\circ$ and $\theta_2 = +90^\circ$

$$\boxed{B_\infty = \frac{\mu_0}{2\pi}\frac{I}{R}}$$

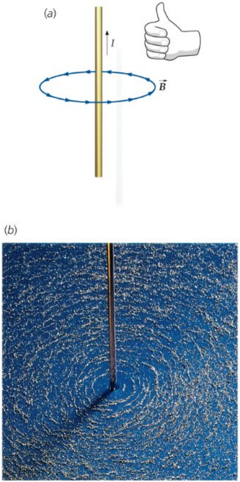


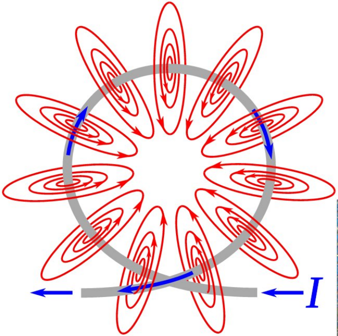
b) Semi-infinitely long:

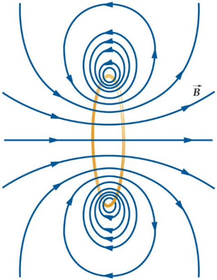
$\theta_1 = 0^\circ$ and $\theta_2 = +90^\circ$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{I}{R}}$$

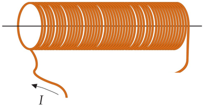
$\vec{B}$: From the wire to the loop

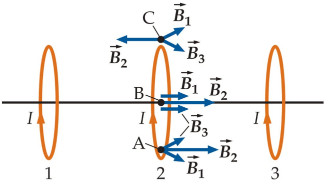






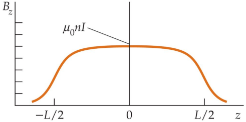
$\vec{B}$: The solenoid



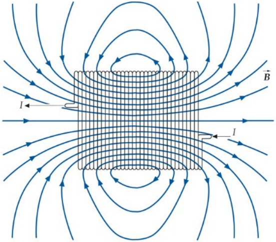


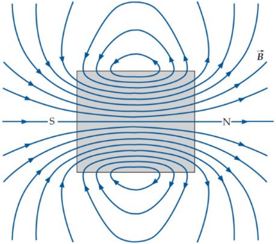
Magnetic field inside a solenoid

$$\boxed{B_z = \mu_0 n I} \rightarrow \text{constant!}$$



$$n = \frac{N}{L} = \frac{\#\text{ of loops}}{\text{length}}$$





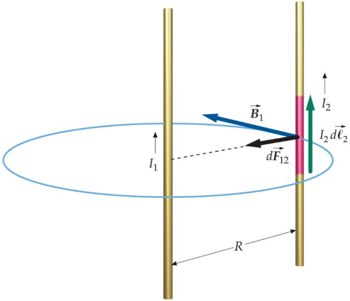
Magnetic Force Between Parallel Wires

$$dF_{12} = |I_2 d\vec{l}_2 \times \vec{B}_1| = I_2 dl_2 B_1$$

$$dF_{12} = I_2 dl_2 \frac{\mu_0 I_1}{2\pi R}$$

force per unit lenght:

$$\frac{dF_{12}}{dl_2} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{R}$$

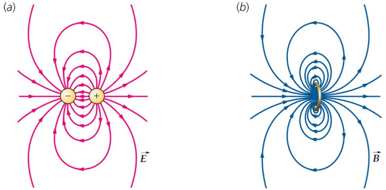


Ampère's Law

Gauss' law turned out to be a powerful tool to calculate electric fields of charged distribution which were highly symmetric ($\oint \vec{E} \cdot d\vec{A} = Q_b/\varepsilon_0$).
A question arises: is there something equivalent for the magnetic field? NO!

$$\oint \vec{B} \cdot d\vec{A} = 0$$

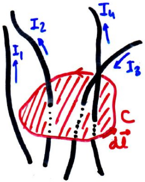
Always, it is useless!



Ampere's law relates the tangential component B_t of the magnetic field summed (integrated) around a closed curve C to the current I_c that passes through any surface bounded by C. It can be used to obtain an expression for $\vec{B}$ in highly symmetric situations:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_C = \mu_0 (I_2 - I_3 + I_4)$$

Ampère's law



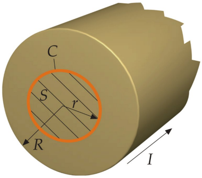
Ampère's Law $\left(\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_C\right)$ applications

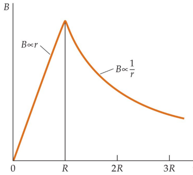
a) $\vec{B}$ inside and outside a wire

$$\oint_C \vec{B}\cdot d\vec{l} = B\oint dl = B2\pi r = \mu_0 I_C \Rightarrow B = \frac{\mu_0 I_C}{2\pi r}$$

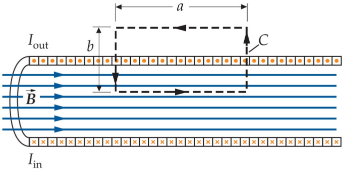
$\boxed{r \geq R}$ $\quad I_C = I \rightarrow B = \frac{\mu_0 I}{2\pi r}$

$\boxed{r \leq R}$ $\quad \frac{I_C}{\pi r^2} = \frac{I}{\pi R^2} \rightarrow B = \frac{\mu_0 I}{2\pi R^2} r$





a) $\vec{B}$ inside and outside a solenoid



$$\oint_C \vec{B}\cdot d\vec{l} = \int_a B dl = Ba = \mu_0 I_C \quad ¿I_C? \quad n = \frac{N}{L} = \frac{\#\text{ of loops}}{\text{length}} \rightarrow I_C = Ina$$

$$\boxed{B = \mu_0 In = \mu_0 IN/L}$$