10. Electromagnetic induction

Tipler and Mosca chapters: 28 and 29

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Introduction

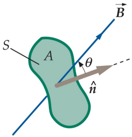
- Oersted showed that an electric current produces a magnetic field. $I \Rightarrow \vec{B}$
  Is the opposite also true? i.e. $I \Leftarrow \vec{B}$

In the early 1830s, Michael Faraday in England and Joseph Henry in the USA independently discovered that in a changing magnetic flux through a surface bounded by a closed stationary loop of wire induces a current in the wire. The emfs and currents caused by such changing magnetic fluxes are called induced emfs and induced currents. The process itself is referred to as induction. Faraday and Henry also discovered that in a static magnetic field a changing magnetic flux through a surface bounded by a moving loop of wire induces an emf in the wire.

Magnetic flux

The magnetic flux through the circuit refers to the flux of the magnetic field through any surface bounded by the circuit:

$$\phi_m = \int_S \vec{B} \cdot d\vec{A} \qquad \text{(remember that} \quad \phi_m = \oint_S \vec{B} \cdot d\vec{A} = 0 \quad \text{always!)}$$



If the surface is flat with area A, and $\vec{B}$ is uniform (has the same value) over the hole surface:

$$\phi_m = \vec{B} \cdot \vec{A} = BA \cos\theta$$

For coils with N turns, remember to multiply the flux through a coil with N !
$\phi_m = N\vec{B} \cdot \vec{A} = NBA \cos\theta$

Faraday's law

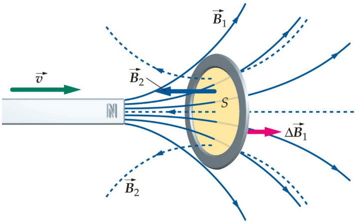
Experiments by Faraday and Henry showed that if the magnetic flux through a surface bounded by a circuit is changed by any means, an emf equal in magnitude to the rate of change of the flux is induced in the circuit. We usually detect the emf by observing a current in the circuit, but the emf is present even if the circuit is nonexistent or incomplete (not closed) and there is no current:

$$\boxed{\varepsilon = I_{\text{ind}} R = -\frac{d\phi_m}{dt}} \qquad \varepsilon = \oint_C \vec{E} d\vec{l} = -\frac{d}{dt}\int_S \vec{B} d\vec{A}$$

Faraday's law

Lenz's law

The negative sign in Faraday's law has to do with the direction of the induced emf. The induced emf (or current) is in such a direction as to oppose, or tend to oppose, the change that produces it.

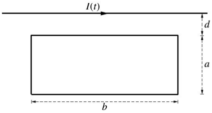


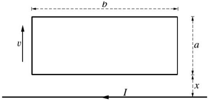
Possible mechanisms to induce current in a circuit

According to Faraday, the value of the magnetic flux through our loop has to change through time to produce a current. What are the options?

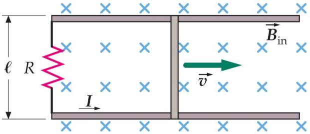
$$\phi_m(t) \approx \vec{B} \cdot \vec{A} = BA\cos\theta$$

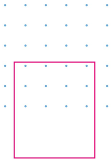
- $B(t)$



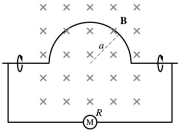


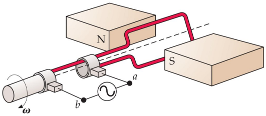
- $A(t)$





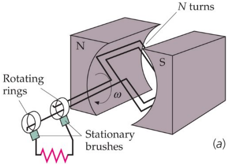
- $\theta(t)$ Alternating current

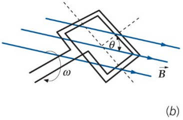


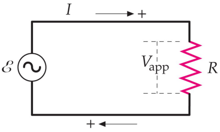


Alternating current

A simple AC generator that consists of a coil of area A and N turns rotating in a uniform magnetic field







$$\phi_m = NBA\cos\theta(t) \quad \text{where} \quad \theta = \omega t + \theta_0 \rightarrow \phi_m = NBA\cos(\omega t + \theta_0)$$

$$\varepsilon = -\frac{d\phi_m}{dt} = NBA\omega\sin(\omega t + \theta_0)$$

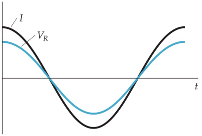
Alternating current within a circuit with a resistor

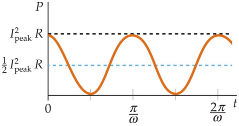
For: $\theta_0 = \pi/2$ and $V_{\text{peak}} \equiv NBA\omega$

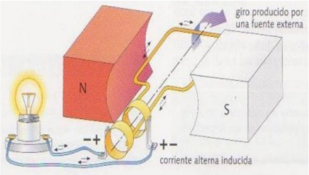
$$V_R = \varepsilon = IR = V_{\text{peak}}\cos(\omega t)$$

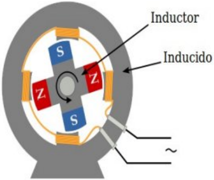
$$I = \frac{V_{\text{peak}}}{R}\cos(\omega t)$$

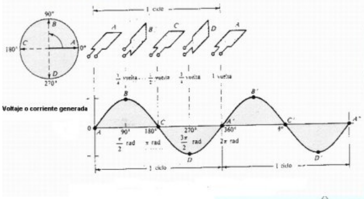
$$P = I^2R = I_{\text{peak}}^2 R\cos^2(\omega t)$$

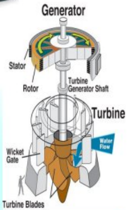


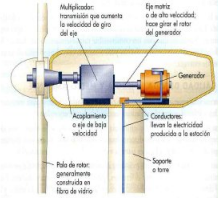


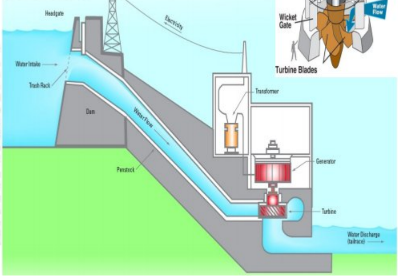

giro producido por
una fuente externa
N
S
-+
+-
corriente alterna inducida


Inductor
Inducido
S
Z
Z
S
~


1 ciclo
90°
180°
0°
270°
Voltaje o corriente generada
0
90°
180°
270°
360°
rad
π rad
rad
2π rad
1 ciclo
1 ciclo


Generator
Stator
Rotor
Turbine
Generator Shaft
Turbine
Wicket
Gate
Water
Flow
Turbine Blades


Multiplicador:
transmisión que aumenta
la velocidad de giro
del eje
Eje motriz
o de alta velocidad;
hace girar el rotor
del generador
Generador
Acoplamiento
o eje de baja
velocidad
Conductores:
llevan la electricidad
producida a la estación
Pala de rotor:
generalmente
construida en
fibra de vidrio
Soporte
o torre


Headgate
Electricity
Water Intake
Trash Rack
Dam
Water Flow
Transformer
Penstock
Generator
Turbine
Water Discharge
(tailrace)

Inductance

Self-inductance

Consider a coil carrying a current I. This will produce a magnetic field $\vec{B}$ everywhere in space that varies from point to point (not uniform in general), but the value of $\vec{B}$ at each point will be proportional to I. In other words, the bigger the current the stronger the field it produces. One can define the magnetic flux through the same coil producing $\vec{B}$ and will therefore also be proportional to I:

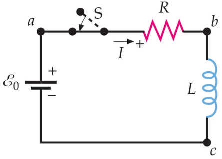
$\boxed{\phi_m = LI}$ L is called self-inductance. $(1$ henry $H = 1\frac{Wb}{A})$

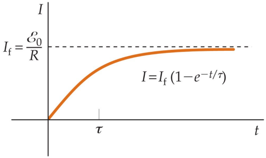
Example: The self-inductance of a solenoid can be analytically calculated

$$B = \mu_0 \frac{N}{l} I$$

$$\phi_m = \mu_0 \frac{N}{l} IAN \rightarrow L = \frac{\phi_m}{I} = \mu_0 n^2 Al$$

RL circuit and Magnetic Energy





Kirchoff's rule:

$$\varepsilon_0 - IR - L\frac{dI}{dt} = 0 \rightarrow \varepsilon_0 I = I^2 R + LI\frac{dI}{dt}$$

$\varepsilon_0 I$ is the electrical power delivered by the battery to the circuit.
$I^2 R$ is the power consumed by the resistor.
$LI\frac{dI}{dt}$ is the power delivered to the inductor.

$$\frac{dU_m}{dt} = LI\frac{dI}{dt} \rightarrow dU_m = LIdI \rightarrow U_m = \int_0^{I_f} LIdI = \frac{1}{2}LI_f^2$$

$$U_m = \frac{1}{2}LI^2 \quad \text{Energy stored in an inductor}$$

For the particular case of a solenoid ($B = \mu_0 nI$ and $L = \mu_0 n^2 Al$):

$U_m = \frac{B^2}{2\mu_0}Al$ the magnetic energy density $\boxed{u_m = \frac{B^2}{2\mu_0}}$