1. Gaia: Magnitudeak, Unitateak eta Bektoreak

Tipler eta Mosca:	1 kapitulua
Fisika Orokorra:	1 eta 2 kapituluak

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Magnitudeak eta Unitateak

Fisikaren helburua Natura aztertzea da, elkarrekintzak ulertzea. Horretarako, natura ondo deskribatzen dituzten **magnitudeak** definitu behar dira.

- OINARRIZKOAK (SI)
  - Luzera: L (m)
  - Denbora: T (s)
  - Masa: M (kg)
  - Tenperatura: K (K)
  - Korronte elektrikoa: A (A)
  - Argi-intentsitatea: I (cd)
- ERATORRIAK
  - Abiadura: LT^{-1} (m/s)
  - Indarra: MLT^{-2} (N)
  - Momentu lineala: MLT^{-1}

Table 1-2 Dimensions of Physical Quantities

Quantity	Symbol	Dimension
Area	A	L^2
Volume	V	L^3
Speed	v	L/T
Acceleration	a	L/T^2
Force	F	ML/T^2
Pressure (F/A)	p	M/LT^2
Density (M/V)	ρ	M/L^3
Energy	E	ML^2/T^2
Power (E/T)	P	ML^2/T^3

Zenbait magnitude orden

Table 1-1 Prefixes for Powers of 10*

Multiple	Prefix	Abbreviation
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Table 1-3 The Universe by Orders of Magnitude

Size or Distance	(m)	Mass	(kg)	Time Interval	(s)
Proton	10^{-15}	Electron	10^{-30}	Time for light to cross nucleus	10^{-23}
Atom	10^{-10}	Proton	10^{-27}	Period of visible light radiation	10^{-15}
Virus	10^{-7}	Amino acid	10^{-25}	Period of microwaves	10^{-10}
Giant amoeba	10^{-4}	Hemoglobin	10^{-22}	Half-life of muon	10^{-6}
Walnut	10^{-2}	Flu virus	10^{-19}	Period of highest audible sound	10^{-4}
Human being	10^{0}	Giant amoeba	10^{-8}	Period of human heartbeat	10^{0}
Highest mountain	10^{4}	Raindrop	10^{-6}	Half-life of free neutron	10^{3}
Earth	10^{7}	Ant	10^{-4}	Period of Earth's rotation	10^{3}
Sun	10^{9}	Human being	10^{2}	Period of Earth's revolution around the Sun	10^{7}
Distance from Earth to the Sun	10^{11}	Saturn V rocket	10^{6}		
		Pyramid	10^{10}	Lifetime of human being	10^{9}
Solar system	10^{13}	Earth	10^{24}	Half-life of plutonium-239	10^{12}
Distance to nearest star	10^{16}	Sun	10^{30}	Lifetime of mountain range	10^{15}
Milky Way galaxy	10^{21}	Milky Way galaxy	10^{41}	Age of Earth	10^{17}
Visible universe	10^{26}	Universe	10^{52}	Age of universe	10^{18}

Dimentsio ekuazioak

Edozein legek magnitudeen dimentsioak errespetatu behar ditu!

$$v = \frac{1}{2}at^2 \longrightarrow [v] = \frac{1}{2}(\frac{L}{T^2})T^2 = L\,???$$

- Adibidea: Kalkulatu G konstante grabitatorioaren dimentsioak eta unitateak. $F = G\frac{m_1 m_2}{r^2}$
- Adibidea: Ondorengo ekuazioetan, x metrotan dago, t segundotan eta v metro zati segundutan. Zeintzuk dira C_1 eta C_2 konstanteen unitateak (SI)?

$$\text{a)} x = C_1 + C_2 t$$

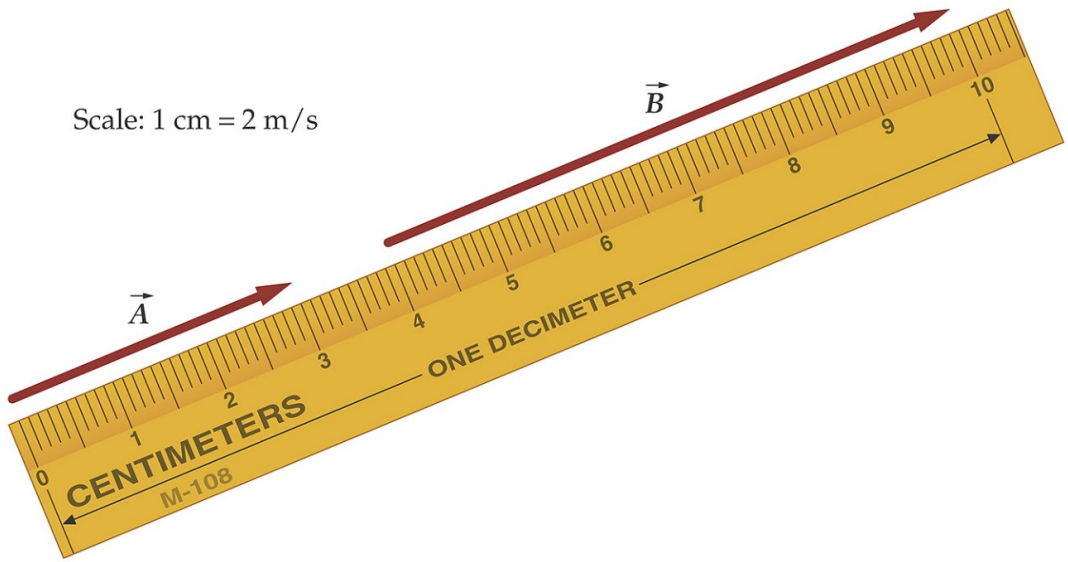
$$\text{b)} v^2 = 2C_1 x$$

$$\text{c)} x = C_1 \cos C_2 t$$

Bektoreak

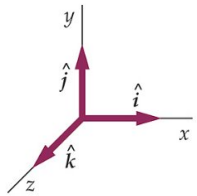
Orokorrean 2 magnitude mota erabiliko ditugu:

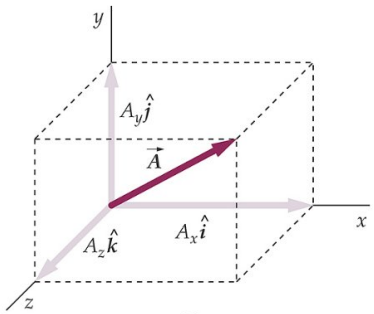
- **Eskalarrak**: zenbaki (+unitatea) batekin guztiz deskribatuta geratzen direnak. (Tenperatura, masa, dentsitatea...)
- **Bektorialak**: zenbakiaz (+unitatea) gain, informazio gehiago eman behar da $\rightarrow$ norabidea eta noranzkoa. (Abiadura, posizioa, indarra...)

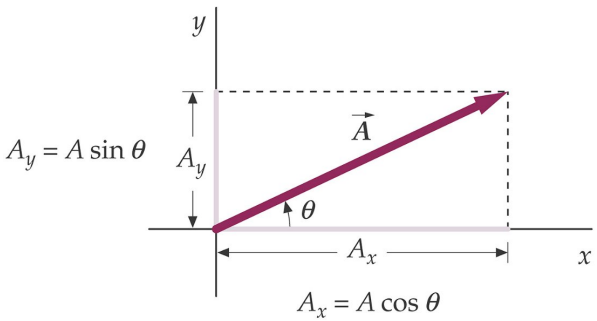


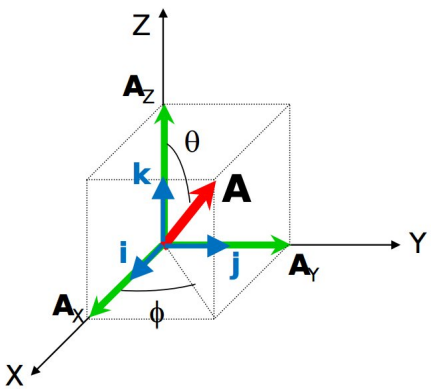
$\vec{A}$ eta $\vec{B}$ bektoreek 6 m/s eta 12 m/s ko moduluak dituzte hurrenez hurren.

Bektoreak









- Erreferentzia sistema kartestarra:

$$\vec{A} = (A_x, A_y, A_z) = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\hat{\imath} = (1,0,0) \quad \hat{\jmath} = (0,1,0) \quad \hat{k} = (0,0,1)$$

- Modulua (geziaren neurria)

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Osagaiak ezagunak izan beharrean, bektorearen modulua eta inklinazio angelua badakigu:

$$A_x = A\sin\theta$$
$$A_y = A\cos\theta$$

3 dimentsiotan:

$$A_x = A\sin\theta\cos\phi$$
$$A_y = A\sin\theta\sin\phi$$
$$A_z = A\cos\theta$$

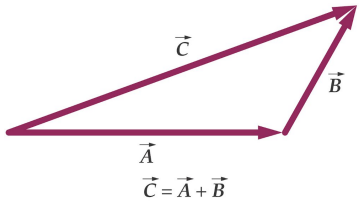
Aljebra bektoriala

Bektoreak definituta ditugu, orain, nola operatu dezakegu beraiekin?

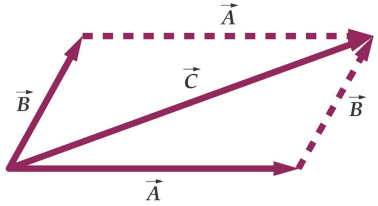
- Batuketa: $\vec{A} + \vec{B}$
- Kenketa: $\vec{A} - \vec{B}$
- Biderketa
  - Eskalarra: $\vec{A} \cdot \vec{B}$
  - Bektoriala: $\vec{A} \times \vec{B}$

Aljebra bektoriala: batuketa

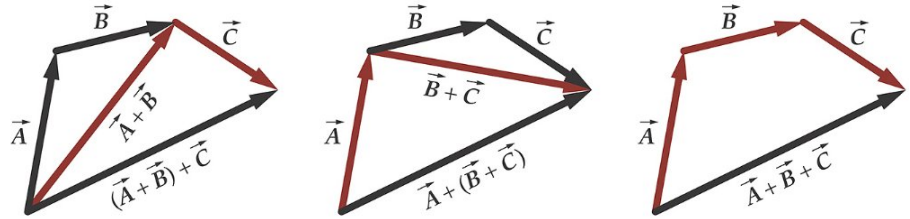
- Grafikoki:



Irudia: jarraipen metodoa



Irudia: paralelogramoaren metodoa



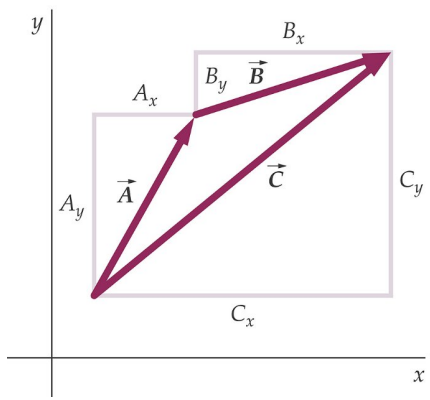
Irudia: Propietate trukakorra. $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

- Matematikoki:

$$\vec{A} = (A_x, A_y, A_z) = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = (B_x, B_y, A_z) = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

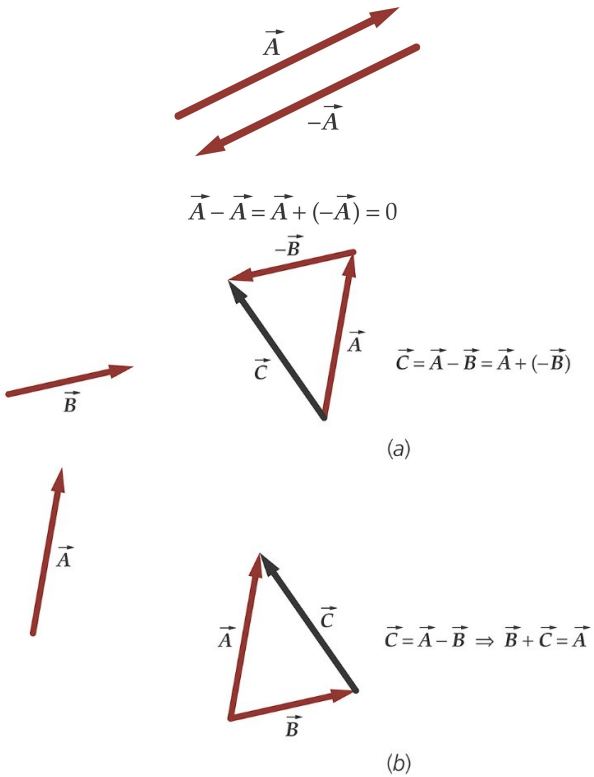
$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_x, A_y + B_y, A_z + B_z) = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$



Aljebra bektoriala

Aljebra bektoriala: kenketa

- Grafikoki:



- Matematikoki:

$$\vec{A} = (A_x, A_y, A_z) = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = (B_x, B_y, A_z) = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{C} = \vec{A} - \vec{B} = (A_x - B_x, A_y - B_y, A_z - B_z)$$
$$= (A_x - B_x)\hat{\imath} + (A_y - B_y)\hat{\jmath} + (A_z - B_z)\hat{k}$$

- Adibidea: $\vec{A} = (2, 0)$ eta $\vec{B} = (-1, -1)$ izanik, batu eta kendu itzazu irudia eginez. Kalkulatu $|\vec{A}|$ eta $|\vec{B}|$

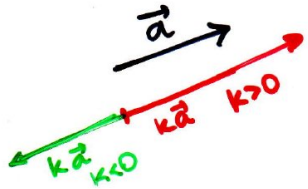
Oharra: N bektore izanez gero, osagaiak banan-banan batu eta kendu behar dira bakoitza bere ikurrarekin:

$$\vec{v} = \vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \ldots + \vec{v}_n = \sum_{i=1}^{n} \vec{v}_i$$

$$\vec{v} = (v_{1x} + \ldots + v_{nx},\ v_{1y} + \ldots + v_{ny},\ v_{1z} + \ldots + v_{nz}) = (v_x, v_y, v_z)$$

Aljebra bektoriala: biderketa eskalarra

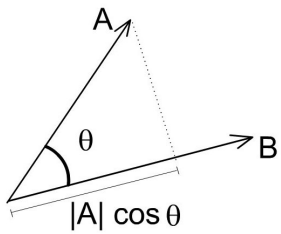
- Bektorea bider eskalarra: luzatu, laburtu ala buelta eman.



$$k \cdot \vec{A} = (kA_x, kA_y, kA_z)$$

Oharra: Nola lortu bektore orokor bati dagokion unitarioa? $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

- Bi bektoreen arteko biderketa eskalarra: $(\vec{A} \cdot \vec{B})$



$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = |\vec{A}||\vec{B}| \cos\theta$$

Zenbaki bat da!!

Propietateak:

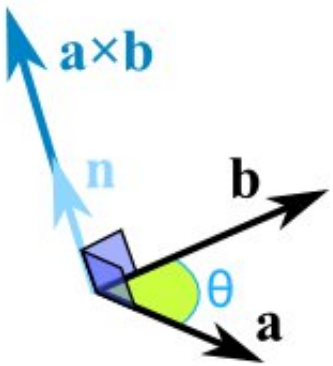
1) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
2) $(\vec{A} + \vec{B}) \cdot \vec{C} = \vec{A} \cdot \vec{C} + \vec{B} \cdot \vec{C}$
3) $\vec{A} \cdot \vec{A} = A^2$ (Moduluaren karratua)
4) $\vec{A} \cdot \vec{B} = 0 \quad \vec{A} \neq 0$ eta $\vec{B} \neq 0 \Rightarrow \vec{A} \perp \vec{B}$
5) $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$
6) $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$
7) $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|}$

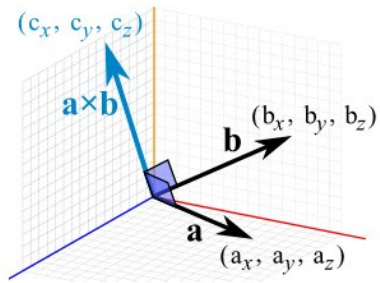
- Adibidea: $\vec{A} = (2, 2)$ eta $\vec{B} = (3, 0)$ izanik, Kalkulatu bi bektoreen arteko angelua. Halaber, kalkula itzazu $\hat{A}$ eta $\hat{B}$

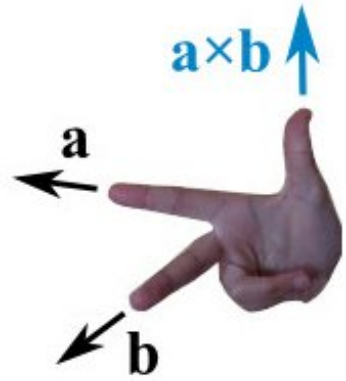
Aljebra bektoriala: biderketa bektoriala

Bi bektoreen arteko biderketa bektoriala: $(\vec{a} \times \vec{b})$ beste bektore bat!!

- Grafikoki: $|\vec{c}| = |\vec{a}||\vec{b}| \sin\theta$







- Matematikoki:

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$$

Propietateak:

1) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
2) $(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$
3) $(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$
4) $\vec{a} \times \vec{b} = 0 \quad \vec{a} \neq 0$ eta $\vec{b} \neq 0 \Rightarrow \vec{a} \parallel \vec{b}$
5) $\hat{\imath} \times \hat{\jmath} = \hat{k} \quad \hat{\jmath} \times \hat{k} = \hat{\imath} \quad \hat{k} \times \hat{\imath} = \hat{\jmath}$
6) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$
7) $|\vec{a} \times \vec{b}|$ = paralelogramoaren azalera

Aljebra bektoriala: biderketa nahasia

- Hiru bektoreen arteko biderkadura mixtoa:

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \text{zenbaki bat}$$

Irudiko paralelepipedoaren bolumena adierazten du zenbakiak.